GRADE 12

MATHEMATICS

MODULE 1: INTRODUCTION TO EXPONENTS AND SURDS

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About eCUBE ONLINE

eCUBE ONLINE is the new online extension of E-SQUARE EDUCATION.

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- Online assistance with choices of subjects based on previous results and career paths.
- Free GeniusU testing.
- No separate resources required, such as guidelines, textbooks and separate assessment tools.
- Sample learning material (first Subject module) is available to view before registration and payment.
- Learning fee includes examination fee.

Availability of free mentor service

- Three (3) hours per subject mentorship for free.
- Students can communicate with their mentor via zoom, email, or WhatsApp or telephone.
- Students may also comment/pose questions on the Special Request section on the learning site that is screened and answered by subject experts. This Special Request section will be accessible to all students, so could provide answers for students who might have had the same enquiry.

User-friendly learning format

- Each matric subject is divided into 12 modules to ensure paced and easy learning.
- You have access to learning material, 24 hours per day and 7 days a week.
- Monitor your progress at the end of each module.
- Each module has exercises based on the topics covered in the module and previous module.
- The questions are based on the type of assessment candidates may expect in the National examination to practice the application of knowledge gained.
- At the end of each module, a compulsory quiz ensures that the candidate has gained the general knowledge required for the topic covered before progress is made to the following module.
- The modules were compiled from multiple resources, both prescribed by the Department of Education and other professionals, to ensure that the topics are covered in detail and from all perspectives.
- Subject specialists with years of experience in teaching their subjects, proof-read all modules and assisted with recommendations to ensure full coverage and easy learning.
- Modules are updated as the curriculum changes to ensure the validity of the learning material.

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You should spend more or less 7 hours on this module.

1. FORMAL EXAMINABLE TOPICS FOR MATHEMATICS

During the final examination you will be tested on the following topics:

- Functions
- Number patterns, sequences and series
- Finance, growth and decay
- Algebra
- Differential calculus
- Probability
- Euclidian Geometry and measurement
- Trigonometry
- Analytical Geometry
- Statistics

The format of the two question papers that have to be completed successfully:

Paper	Topics	Duration	Total
1	Patterns and sequences Finance, growth and decay Functions and graphs Algebra, equations, inequalities Differential calculus Probability	3 hours	150
2	Euclidian Geometry Analytical Geometry Statistics and regression Trigonometry	3 hours	150

2. THE NUMBER SYSTEM

To understand exponents and surds, you need to study the number system thoroughly.

2.1. Real Numbers

The numbers that we work with every day are called real numbers. These are divided into the following:

Natural Numbers
$\mathbb{N}$ = {1; 2; 3; …} (positive whole numbers)

Whole Numbers
$\mathbb{N}_0$ = {0; 1; 2; …} (Natural numbers and 0). These numbers do not have any fractional parts and are greater than or equal to 0. For example: 1, 7, 46, 108.

Integers
$\mathbb{Z}$ = {… ;–3; –2; –1; 0; 1; 2; 3; …} These are whole numbers that can be positive, negative or zero.

Rational Numbers
A rational number is a Real number which can be written in the form $\frac{a}{b}$ where a, $b \in Z$ and $b \neq 0$. The rational numbers include all the integers.

- Note
  All terminating, recurring decimals are rational numbers examples: 0. 3; 2,71 ; 5,321784571.

Irrational Numbers
Irrational numbers are numbers that cannot be written as fractions. All decimals that do not terminate or recur are irrational. They have decimals that continue indefinitely with no pattern.

- Look at these numbers on a calculator - $\sqrt{5} = 2{,}23606$ ...pi (π) = 3,141592…
- The calculator will round them off. However, they continue indefinitely without a pattern.
- The symbol for the irrational numbers is $\mathbb{Q}'$, which means the complement of $\mathbb{Q}$ or not $\mathbb{Q}$.

Real Numbers
The set of real numbers, $\mathbb{R}$, is the set of all rational and irrational numbers together. We can also write $\mathbb{R} = \mathbb{Q} \cup \mathbb{Q}'$.

2.2. Non-real numbers

The square root (or any even root) of a negative number, is a non-real number.

$\sqrt{-25}$ is a non-real number.

$\sqrt[4]{-100}$ is a non-real number.

$\sqrt[6]{-120}$ is a non-real number.

2.3. Surds

All square roots, cube roots, etc. that are not rational are called surds.
$\sqrt{2}$; $\sqrt{3}$; $\sqrt{5}$; $\sqrt{6;}$ $\sqrt{7;}$ $\sqrt{8}$ are all surds.

Surds Definition:
Surds are the square roots (√) of numbers which do not simplify into a whole or rational number. It cannot be accurately represented in a fraction. In other words, a surd is a root of the whole number that has an irrational value. Consider an example,
√2 ≈ 1.414213, but it is more accurate to leave it as a surd √2.

Types of Surds

The different types of surds are as follows:

- Simple Surds: A surd that has only one term is called simple surd. Example: √2, √5, …
- Pure Surds: Surds which are wholly irrational. Example: √3
- Similar Surds: The surds having the same common surds factor
- Mixed Surds: Surds that are not wholly irrational and can be expressed as a product of a rational number and an irrational number
- Compound Surds: An expression which is the addition or subtraction of two or more surds
- Binomial Surds: A surd that is made of two other surds

Six Rules of Surds

Rule 1

$\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$

Example:

To simplify √18

18 = 9 x 2 = 3^2 x 2, since 9 is the greatest perfect square factor of 18.

Therefore, √18 = $\sqrt{3^2 \times 2}$

= $\sqrt{3^2}$ x √2

= 3 √2

Rule 2

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Example:

√12 / 121 = √12 / √121

$\sqrt{2^2}$ x 3 / 11

Since 4 is the perfect square of 12

$\sqrt{2^2}$ x √3 / 11

= 2√3 / 11

Rule 3

$\frac{b}{\sqrt{a}} = \frac{b}{\sqrt{a}} \times \frac{\sqrt{a}}{\sqrt{a}} = b\frac{a}{\sqrt{a}}$

You can rationalise the denominator by multiplying the numerator and denominator by the denominator.

Example:

Rationalise:

5/√7

5/√7 = 5/√7 x √7/√7

Multiply numerator and denominator by √7

= 5√7/7

Rule 4

$a\sqrt{c} \pm b\sqrt{c} = (a \pm b)\sqrt{c}$

Example:

To simplify,

5√6 + 4√6

5√6 + 4√6 = (5 + 4) √6

by the rule

= 9√6

Rule 5

$$\frac{c}{a + b\sqrt{n}}$$

Multiply top and bottom by a-b √n

This rule enables us to rationalise the denominator.

Example:

To Rationalise:

$$\frac{\mathbf{3}}{\mathbf{2} \times \sqrt{\mathbf{2}}} = \frac{\mathbf{3}}{\mathbf{2} \times \sqrt{\mathbf{2}}} \times \frac{\mathbf{2} - \sqrt{\mathbf{2}}}{\mathbf{2} - \sqrt{\mathbf{2}}} = \frac{\mathbf{6} - \mathbf{3}\sqrt{\mathbf{2}}}{\mathbf{4} - \mathbf{2}} = \frac{\mathbf{6} - \mathbf{3}\sqrt{\mathbf{2}}}{\mathbf{2}}$$

Rule 6:

$$\frac{c}{a - b\sqrt{n}}$$

This rule enables you to rationalise the denominator.

Multiply top and bottom by a + b√n

Example:

To Rationalise:

$$\frac{3}{2-\sqrt{2}} = \frac{3}{2-\sqrt{2}} \times \frac{2+\sqrt{2}}{2+\sqrt{2}} = \frac{6 \times 3\sqrt{2}}{4-2} = \frac{6 \times 3\sqrt{2}}{2}$$

Surds are real numbers which, when expressed as decimals, are non-recurring and non-terminating.

We can work out where a surd lies between two integers on a number line.

How to Solve Surds?

You need to follow some rules to solve expressions that involve surds. One method is to rationalise the denominators and this is done by ejecting the surd in the denominator. Sometimes it may be mandatory to find the greatest perfect square factor to solve surds. This is done by considering any possible factors of the value that is square rooted.

For example, you need to solve for the square root of 144. 2 x 72 gives 144 and we can have a square root of 144 without a surd. Therefore, we say that 144 is the greatest perfect square factor since we cannot take the square root of a bigger number that can be multiplied by another to give 144.

2.4. Rationalising denominators

When a fraction contains a surd in the denominator (the number at the bottom of a fraction), you can change the denominator to a rational number.

This is called ‘rationalizing the denominator’.

If you multiply the numerator (the number at the top of a fraction) and the denominator by the same surd, you are not changing the value of the number.

You are multiplying by 1 $(\frac{\sqrt{2}}{\sqrt{2}} = 1)$

EXERCISE 1

Simplify the following without using a calculator

1. $\sqrt{8} \times \sqrt{2}$
2. $\sqrt[3]{4} \times \sqrt[3]{2}$
3. $\frac{9+\sqrt{45}}{3}$
4. $(2+\sqrt{5})\,(2-\sqrt{5})$

Answers to Exercise 1

1. $\sqrt{8} \times \sqrt{8} = \sqrt{8 \times 2} = \sqrt{16} = 4$
2. $\sqrt[3]{4} \times \sqrt[3]{2} = \sqrt[3]{4 \times 2} = \sqrt[3]{8} = 2$
3. $\frac{9+\sqrt{45}}{3} = \frac{9+3\sqrt{5}}{3} = \frac{3(3+\sqrt{5}}{3} = 3 + \sqrt{5}$
4. $\left(2+\sqrt{5}\right)\left(2-\sqrt{5}\right)$

   $= 2 \times 2 - \sqrt{5} \times \sqrt{5} = 4 - 5 = -1$

EXERCISE 2

Rationalise the following denominators

2.1 $\frac{\sqrt{3}}{\sqrt{2}}$

2.2 $\frac{3}{\sqrt{3}-1}$

Answers to Exercise 2

2.1 $\frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{3}\times\sqrt{2}}{2} = \frac{\sqrt{6}}{2}$

2.2 $\frac{3}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{3(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{3\sqrt{3}+3}{3+\sqrt{3}-\sqrt{3}-1} = \frac{3\sqrt{3}+3}{2}$

EXERCISE 3

3.1 Complete the table for each number

Number	Non-real number	Real number $\mathbb{R}$	Rational number $\mathbb{Q}$	Irrational number $\mathbb{Q}'$	Integer $\mathbb{Z}$	Whole number $\mathbb{N}_0$	Natural number $\mathbb{N}$
13							
5,121212							
$\sqrt{-6}$							
3π							
$\frac{0}{9} = 0$							
$\sqrt{17}$							
$\sqrt[3]{64} = 4$							
$\frac{22}{7}$							

3.2 Identify the following numbers as rational or irrational

a. $\sqrt{16}$ b. $\sqrt{8}$ c. $\sqrt{\frac{9}{4}}$ d. $\sqrt{6\frac{1}{4}}$

e. $\sqrt{47}$ f. $\frac{22}{7}$ g. 0,347347 h. $\pi - (-2)$

i. $2 + \sqrt{2}$ j. 1,121221222

Answers to Exercise 3

3.1

Number	Non-real number	Real number $\mathbb{R}$	Rational number $\mathbb{Q}$	Irrational number $\mathbb{Q}'$	Integer $\mathbb{Z}$	Whole number $\mathbb{N}_0$	Natural number $\mathbb{N}$
13		✓	✓		✓	✓	✓
5,121212		✓	✓				
$\sqrt{-6}$	✓						
3π		✓		✓			
$\frac{0}{9} = 0$		✓	✓		✓	✓	
$\sqrt{17}$		✓		✓			
$\sqrt[3]{64} = 4$		✓	✓		✓	✓	✓
$\frac{22}{7}$		✓	✓				

3.2

a. $\sqrt{16}$ Rational

b. $\sqrt{8}$ Irrational

c. $\sqrt{\frac{9}{4}}$ Rational

d. $\sqrt{6\frac{1}{4}}$ Rational

e. $\sqrt{47}$ Irrational

f. $\frac{22}{7}$ Rational

g. 0,347347 Rational

h. $\pi - (-2)$ Irrational

i. 2 + $\sqrt{2}$ Irrational

j. 1,121221222 Irrational

MATHEMATICAL SIGNS AND SYMBOLS

Symbol	Definition	Symbol	Definition	Symbol	Definition
$+$	Plus; positive	:	Ratio of (6:4)	( )	Parentheses; multiply
$-$	Minus; negative	::	Proportionately equal (1:2::2:4)	$-$	Vinculum; division; chord of circle or length of line
$\pm$	Plus or minus; positive or negative; degree of accuracy	≈; ≏; ≑	Approximately equal to; similar to	$\overrightarrow{AB}$	Vector
$\mp$	Minus or plus; negative or positive	$\cong$	Congruent to; identical with	$\overline{AB}$	Line segment
$\times$	Multiplied (6 × 4)	$>$	Greater than	$\overleftrightarrow{AB}$	Line
$\cdot$	Multiplied (6·4); scalar product of two vectors (A·B)	$\gg$	Much greater than	∞	Infinity

÷	Divided by	≯	Not greater than	n^2	Squared number
/	Divided by; ratio of $(^6/_4)$	<	Less than	n^3	Cubed number
–	Divided by; ratio of $(\frac{6}{4})$	≪	Much less than	$\sqrt[2]{\ }$	Square root
=	Equals	≮	Not less than	$\sqrt[3]{\ }$	Cube root
≠	Not equal to	≧; ≥;	Equal to or greater than	%	Percent
≡	Identical with; congruent to	≦; ≤	Equal to or less than	℃	Degrees
≜	Corresponds to	∝	Directly proportional to	∠	Angle
≚	Equiangular	π	Pi; the ratio of the circumference to the diameter of a circle = 3.14	α	Alpha (unknown angle)
θ	Theta (unknown angle)	∟	Right angle	≑	Parallel
∴	therefore	∵	Because	≞	Measured by

END OF MODULE 1